Nasa CR 65501

FINAL REPORT

VISUAL THREE-VIEW SPACE-FLIGHT SIMULATOR

CONTRACT NAS 9-1375

1 August 1964

PREPARED FOR

National Aeronautic and Space Administration
Manned Spacecraft Center
Houston, Texas

GENERAL ELECTRIC

DEFENSE ELECTRONICS DIVISION • ELECTRONICS LABORATORY • ITHACA, N.Y.
CORNELL UNIVERSITY INDUSTRY RESEARCH PARK • ITHACA • NEW YORK

ABSTRACT

This is the Final Report submitted in accordance with the requirements of NASA Contract NAS9-1375, entitled Visual Contact Analog; Three View Interim Space-Flight Simulator. This report contains a summary of the contract effort and a description of the equipment delivered. An appendix to the report contains the results of a study of round-off errors for the CORDIC algorithm.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

I. SUMMARY

The contract was concerned with the design, fabrication, and assembly of a visual attachment to a space vehicle simulator. The visual attachment consists of a special-purpose digital computer, three color television display units, and an optical system for viewing these displays. The optical system consists of three wide-angle window simulators and a celestial sphere which provides a realistic starfield simulation.

On the basis of inputs giving the attitude and position of the vehicle, the computer generates three perspective views of an idealized environment. These three views are displayed on three television screens and then are optically projected to three wide-angle window simulators. To the observer, the effect is that of looking out through three ports in the vehicle. Through one of these windows the observer may also view the celestial sphere, which simulates the stars as seen from the vehicle.

The displayed environment is idealized. It consists of an unbounded plane surface, called the ground plane and a plane field of stars. The ground plane is textured with cyclic orthogonal patterns. These patterns are variable (within certain constraints) by the user. In addition, the ground plane contains a unique area which has a distinct texture and color. The star field is similarly composed of a cyclic pattern of "constellations." This pattern is also variable.

As an alternative to the ground plane, a bounded plane surface, called the rendezvous surface, can be selected as the environment. The rendezvous surface is textured with patterns similar to those of the ground plane.

The environment (i.e., the surface patterns) is stored in the computer. Inputs inform the computer of the attitude and position of the vehicle in its environment. The computer performs a perspective transformation of the environment onto the planes of the three ports of the vehicle. These three perspective pictures appear on the television displays. The optical system allows these pictures to be viewed with the proper angular relationships, collimated so that the environment appears to be distant from the observer. This is accomplished without undue restriction of the observer's head.

This visual attachment was to be sufficiently flexible so as to allow it to adapt to a variety of simulated vehicles and environments. Therefore, provision was made for varying many of the parameters of the pictures. Some of the parameters that can be set by the user are station (or viewing) point position, picture sight line, angular coverage, texture pattern, texture color and texture scale.

Design effort was in three principal areas: digital computer design, display circuitry design, and optical design. The computer consists of eight units plus two interface units. The system design provided for independent timing and operation of the units. This allowed logical design of the units to proceed in parallel with a minimum of interaction.

Three functions are provided by the display units:

1) Display roll is implemented in the display units by electronically rolling the raster as required.

2) Some distortion exists in the optical system. Precise nonlinear sweeps are generated by the display unit to produce a nonlinear raster pattern. The resulting nonlinear display compensates for the optical distortion.

3) To avoid disturbing moire patterns, fine detail is gradually faded out of the pictures. This fading, in response to information from the digital computer, is accomplished by the display units.

The optical design effort produced two 60-degree window simulators and one 25-degree window simulator. In addition, support structures for the three optical units, which allow the windows to be used in a variety of configurations, were designed. The design of the celestial sphere illumination and drive mechanism was also a part of the optical-mechanical design effort.

Previous reports (Quarterly Reports 1, 2, and 3, and Monthly Reports) contain detailed descriptions of the design effort. The Instruction Manuals for the system provide operation and maintenance information, as well as the theory of operation.

In connection with the digital design effort a study of CORDIC round-off error was conducted. The results of this study are presented in an appendix to this report.

II. DESCRIPTION OF THE SYSTEM

A. General Discussion of the Computation

Each of the views generated by the computer is a perspective picture of several patterned surfaces displayed on a television raster. To generate a perspective picture of a surface, the display raster pattern is first projected onto the surface. That is, a line through the viewing (or station) point and the scanning spot is assumed and its intersection with the surface found. This point of intersection is called the image of the scanning spot in the surface. The location of the image is a function of the attitude and position of the display (and hence of the vehicle in which it is located) with respect to the surface. The pattern on the surface is given as input data and is stored in the computer. When the coordinates of the scanning spot image are determined, they are referred to these stored data (called the map table) to find the color of the surface at this point. The surface color then is used to specify the drive to the electron guns of the cathode ray tube on which the picture is being displayed.

Sychronization for the raster is generated by the computer. Commercial television standards are used. The raster lines are assumed to have no roll; i.e., they are parallel to the surface, so that each raster line is a constant-scale line. Thus if a raster line were divided into equal segments, its image in the surface would also be divided into equal segments. Each raster line is divided into approximately 256 raster elements. Its image therefore consists of 256 collinear segments, all equal in length and direction. It is clear, then, that the entire image of a raster line can be specified by four numbers: two coordinates for the left end of the image of the raster line and two for the north and east components of one of the 256 segments. These numbers will in general be different for each raster line. The computation of these numbers requires rotation of coordinates (expression of the scanning spot position in surface, e.g., earth coordinates), multiplication and division.

The video is generated by scanning the map table in synchronism with the raster. For each video pulse period [approximately 0.2 microsecond (μsec)] the position in the map table must be advanced a distance equal to one segment. Two binary flip-flop registers hold the north and east coordinates of the scanning spot image. At the beginning of a raster line, they are set up with the coordinates of the left end of the raster line image. The map table is entered by applying the outputs of these flip-flops to a decoding network controlled by the map table. The resulting output of the decoding network is the color of the surface at that point. Then, each 0.2 μsec, the north and east coordinate registers are advanced one segment and the color at the new location read out. Each surface appearing in the computer is drawn in this way. In general, the surfaces have much in common and so do not require entirely independent computations.

Initially the raster lines were assumed to be parallel to the surface (i.e., roll is zero). To justify this assumption, the raster is actually rolled on the CRT to a horizontal position. The angle through which it must be rolled is computed on the basis of vehicle attitude inputs.

The Visual Three-View Space-Flight Simulator System consists of a display computer, three interface units, a monitor unit, three TV display units, three optical windows, a celestial sphere, and window support structure. A description of this system, shown in block diagram form in Figure 1, is provided in the following paragraphs.

B. Display Computer

The principal part of the Space-Flight Simulator is a special-purpose digital computer. This computer, housed in the console shown in Figure 2, consists of eight major units, a system control panel, and several auxiliary units. Input data to the computer is supplied via separate interface units. Two Interface Units are provided to convert the NASA PACE and TRICE computer outputs into a form appropriate for the display computer. A third unit, designated the Vehicle Simulator Unit, is provided as a means for supplying auxiliary inputs to the GE display computer in the absence of inputs from the NASA computers.

1. Console Units

a. The Program Control Unit or PCU employs a magnetic core memory to store the computer program and numerical data. It executes the frame by frame (1/30 second) computation in accordance with the stored program.

b. The Coordinate Rotation Unit or CRU specializes in multiplication and transformation of vectors under rotation of coordinates. It receives its data and instructions from the PCU.

c. The Line-by-line Adder and Sync Unit or LASU performs the 15.75-kc cyclic additions which are prominent in the perspective computation. It receives data during each frame period from the PCU. The LASU also provides the synchronization signals which control the raster scans.

d. The Divider Unit or DU is a high-speed divider. It takes the sums from the line-by-line additions and generates quotients at the same rate. These quotients specify the image of the start of the raster line, and the direction and rate at which the image will move as the line is scanned.

e. The Element-by-Element Adder Units or EAU receive the quotients from the DU and follow the movement of the scanning spot image. They provide the coordinates of the scanning spot image at a scan rate of 5 million points per second. There are three EAU's, one for each picture produced.

f. The Maps and Video Processor Unit or MVPU takes the scanning spot image coordinates and determines the color of the environment at these points. It is in the MVPU that the texture patterns are stored. The MVPU also provides timing and control for the EAU's.

2. System Control Panel

Panel controls and indicators are provided for system control, test, and calibration. The system control panel consists of the Power and Mode Selection, Keyboard, Indicator Lights, Cell Size, and Test and Calibration panels.

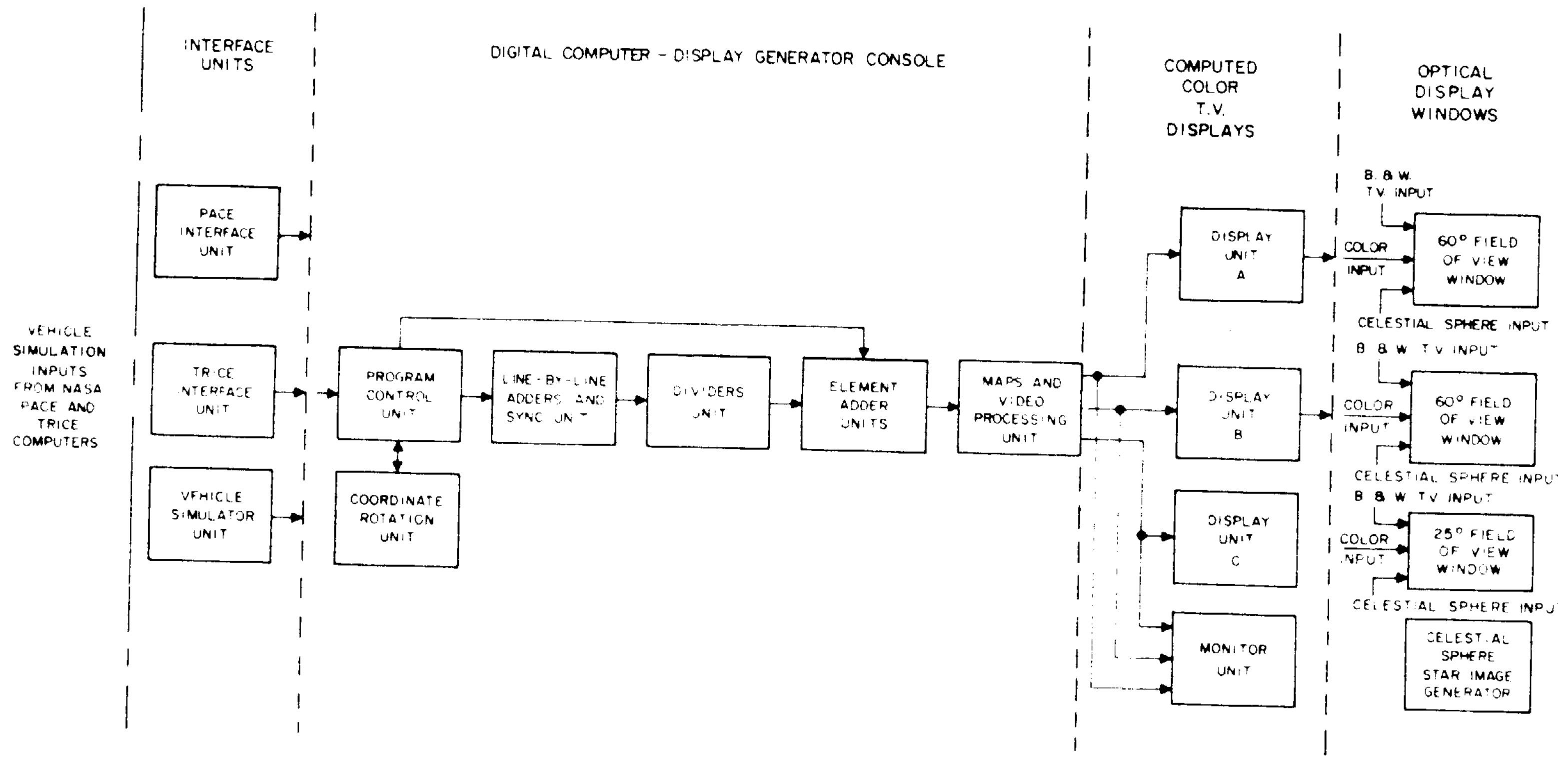

Figure 1. Block Diagram - Visual Three-View Space Flight Simulator

Figure 2. Computed Space-Flight Display Console and Monitor Unit

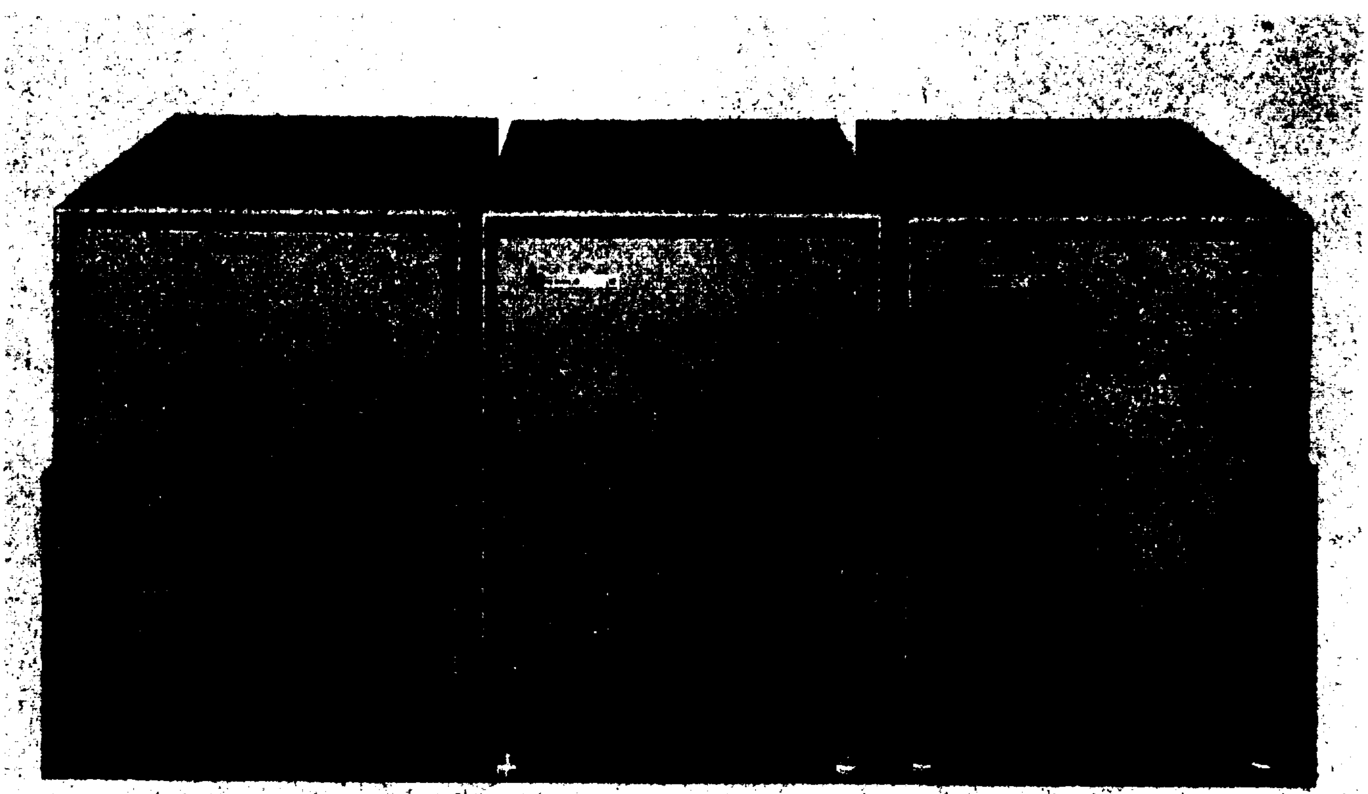

Figure 3. Interface Units

3. Auxiliary Units

A 7-inch black and white television monitor (shown at the right of the console) is provided to facilitate system test and calibration. The console also includes a primary power distribution unit, input-output interconnection unit, and two storage units.

C. Interface Units

The Interface Units (Figure 3) consist of the PACE (1), TRICE (2), and Vehicle Simulator (3) Units. The inputs for the computation are vehicle position and attitude. These inputs are provided by the NASA computers that simulate the vehicle dynamics and provide navigation data. The PACE Interface Unit, or PIU, and TRICE Interface Unit, or TIU, convert the NASA computer outputs into a suitable form for the General Electric Computed Space-Flight Display System. The Vehicle Simulator Unit can be used to provide simulated inputs to the computed display system. The function of these units is as follows:

1) The PACE Interface Unit or PIU accepts dc voltages from the NASA PACE Computer. Upon receipt of a request from the console, it converts the appropriate voltage to a binary number and shifts it to the console.

2) The TRICE Interface Unit or TIU accumulates incremental outputs from the NASA TRICE Computer. Approximately each 1/30 second, it receives a request from the console to transfer each of the accumulated quantities.

3) The Vehicle Simulator Unit or VSU sends thrust or acceleration inputs to the console. These inputs are then processed by a subroutine in the PCU to provide a simplified simulation of a space vehicle.

The Interface Units can be conveniently located near the NASA computers. Changes in the interface are easily accommodated without major changes in the console. Furthermore all data transmission over large distances (hundreds of feet) is performed with digital signals and at low rates, resulting in low noise susceptibility and high reliability.

D. Displays and Monitors

Three color TV Display Units are provided (one of which is shown in Figure 4). Display Units A and B are designed for a ±30-degree view and are referred to as Type I; Display Unit C is designed for a ±12. 5-degree view and referred to as Type II. Each display unit contains a video processor which receives the video information from the MVPU. The video processor provides the necessary amplification and implements the fading operation that makes pattern detail fade out gradually as it becomes small. Each display unit also contains deflection circuitry which provides for raster roll in response to roll inputs and raster nonlinearity required to correct for the flat-face distortion inherent in the CRT and the pincushion distortion introduced by the optical display windows. The power supplies for all three display units are housed in the cabinets shown in Figure 5.

Since the Display Units cannot be seen by the console operator, the three displays are repeated by a Monitor Unit located behind the console (see Figure 2). The Monitor Unit contains three color television monitors and a small computer unit (the Roll Symbol Generator Unit). The three displays are repeated without the raster nonlinearity (since the monitor pictures are to be viewed without the optical system) and without

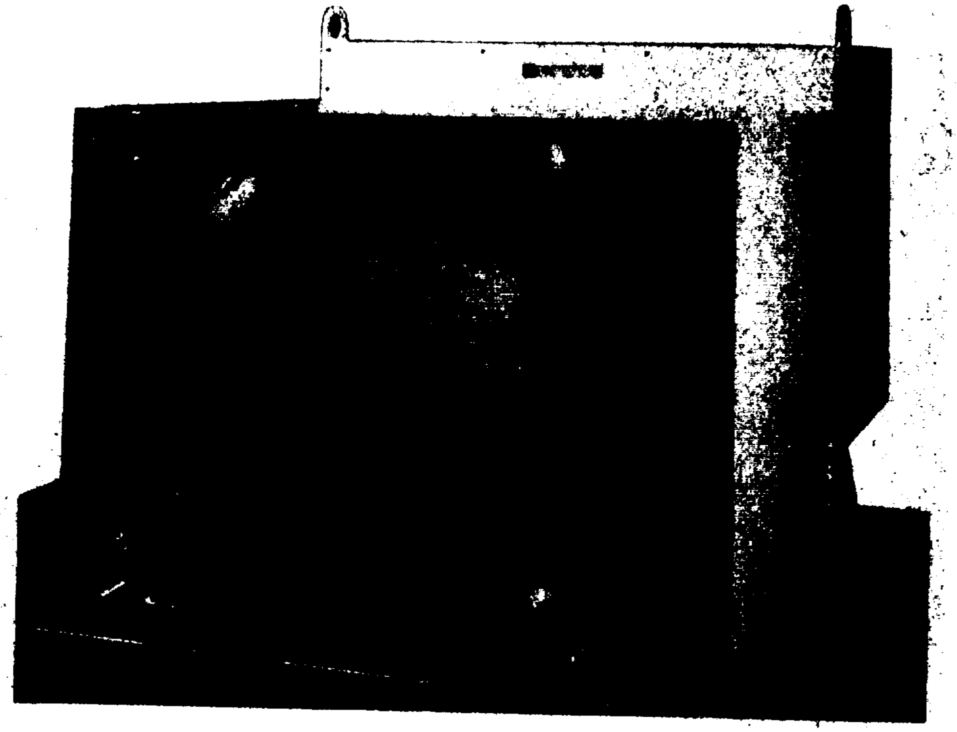

Figure 4. Display Unit (typical)

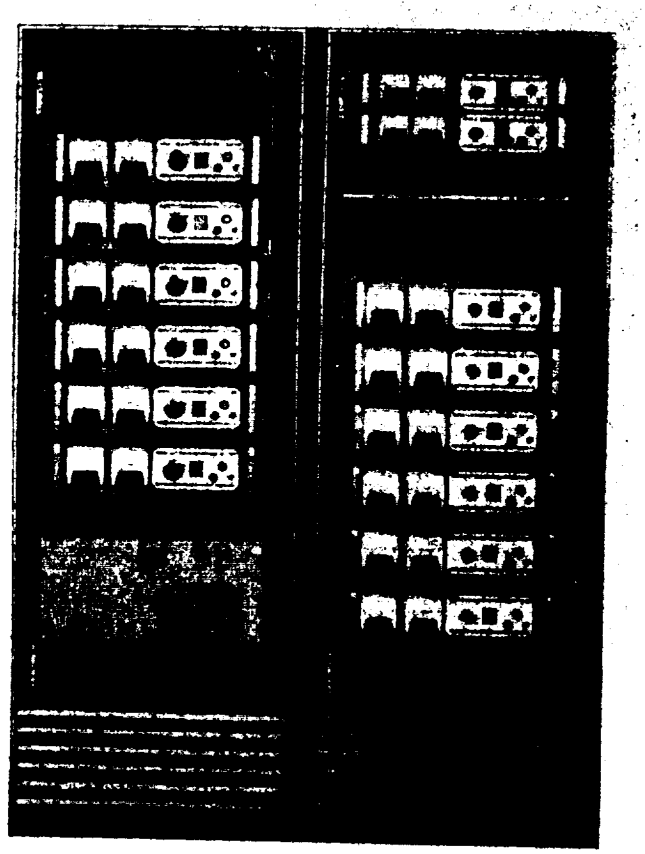

Figure 5. Display Unit Power Supplies

raster roll (for economy). To provide the console operator with roll information, roll symbols are added. These red and green squares may be thought of as port and starboard lights.

E. Window Simulators

1. General

The computed environment generated by the GE Computer is projected through three optical windows, the result being highly realistic images simulating those which would be observed during actual space flight. The abbreviated name commonly used for the three Optical Window Simulators developed and manufactured by the Farrand Optical Company is "array," or "array of three optical window simulators."

The two Type I windows and the Type II window used in the system differ in field-of-view presentation. However, each is capable of providing simulation of the space flight scene with an unprecedented degree of realism. All three windows can provide a view which is the composite of that generated by a 21-inch, high resolution, black-and white TV input, a 21-inch computed display color TV input, and the 27-inch-diameter celestial sphere star image generator. These three image generators may be utilized alone or in combination. If a combination of two or more image generators is being used, the resulting images are superimposed - the separately generated images being focused (apparently) at infinity.

2. Type I Windows

a. <u>Black-and-White TV Image Projection System</u>. The 21-inch black-and-white TV input, the faceplate of which is shown in the upper left corner of the schematic of the optical layout for the Type I window (see Figure 6), is part of an external TV chain which may provide displays of models and other devices, simulating the space capsule environment. The black and white TV chain was not part of this procurement.

b. <u>Color TV Image Projection System.</u> The 21-inch color TV input, the faceplate of which is shown below the black-and-white TV faceplate on Figure 6, accepts the computer-generator display. The field lens projects the illumination comprising the color TV input display image into incidence with the TV field lens beamsplitter; the input reflected from the TV field lens beamsplitter describes the identical focus followed by the black-and-white TV input.

c. <u>Celestial Sphere Star Image Projection System</u>. The 27-inch diameter celestial sphere is mounted in a three-gimbal-axis structure (Figure 7) separate from the rest of the Type I window structural support frame. Axis 1 (celestial pitch) is the axis normal to the plane of the ecliptic; axis 2 (celestial yaw) is an axis perpendicular to axis 1 and thereby contained within the ecliptic plane, and axis 3 (celestial roll) provides the third axis motion for the gimbal system. The system provides the simulated star-studded surround upon which the TV input images may appear in superposition. One celestial sphere has been supplied for the array of the three optical window simulators; this sphere may be mounted interchangeably on the three windows.

d. <u>Celestial Sphere Illumination System, Type I Window</u>. The celestial sphere illumination system generates the cone of illumination for the sphere. A lamp and spherical mirror are used to provide a real image of the illumination source filament to the filament space, thus conserving and increasing the uniformity of

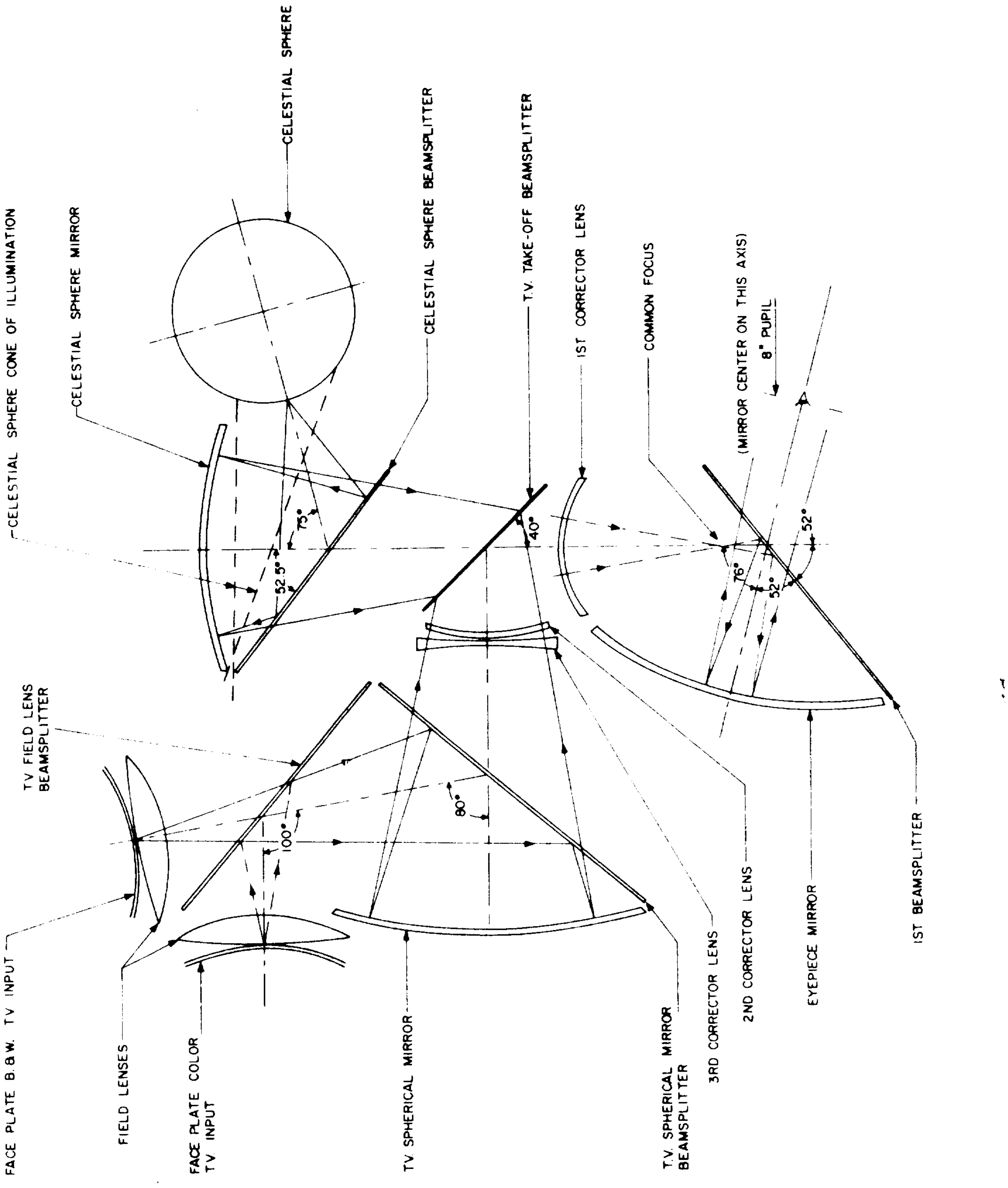

Figure 6. Optical Layout of Type I Windows

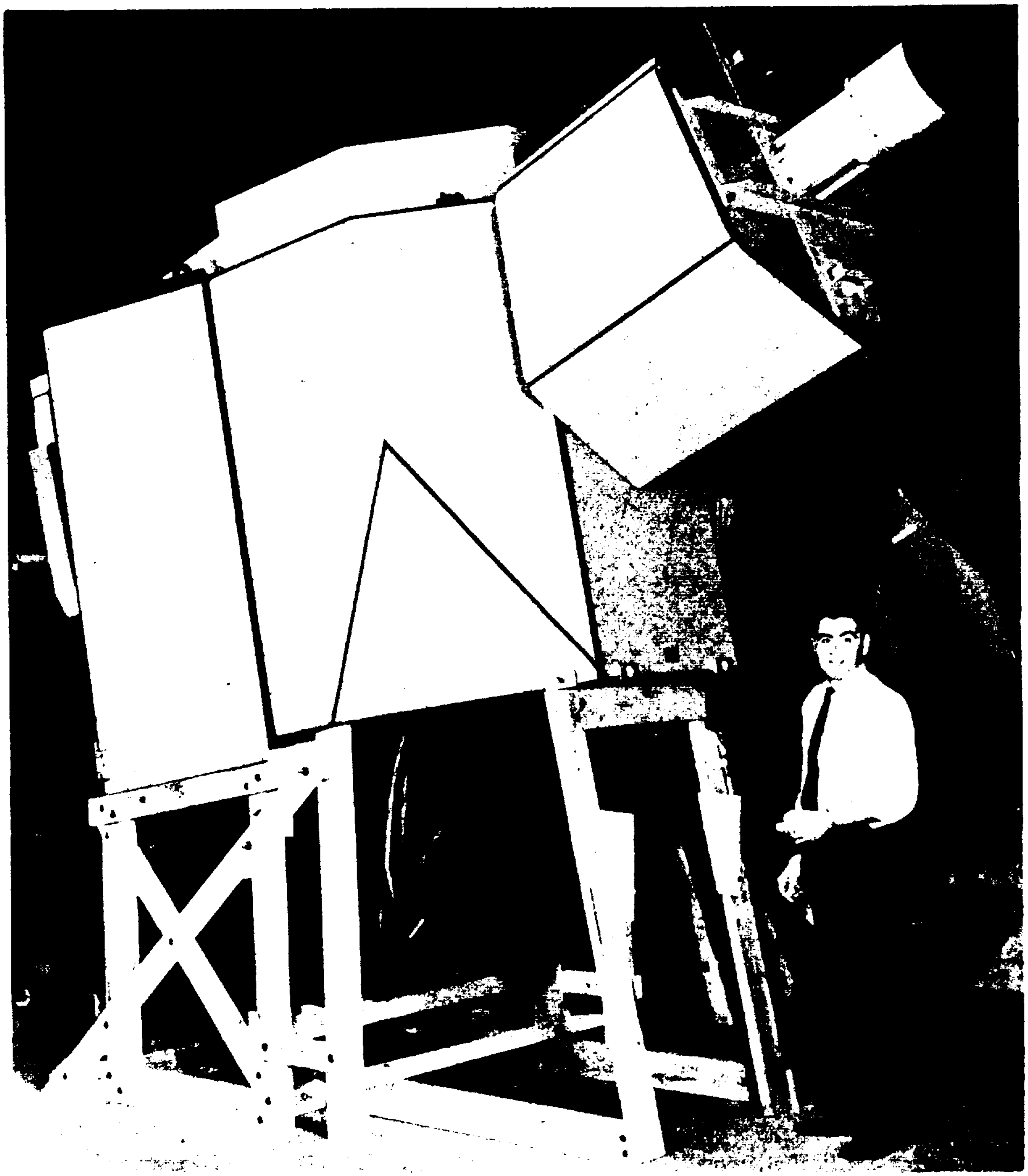

Figure 7. Celestial Sphere Mounted on Structure of Type I Windows

illumination. A condensing system images the filament of the lamp at the aperture of the first field stop; a second condensing system images the point source of illumination at the aperture of the second field stop. This point source of illumination is also illustrated giving rise to the celestial sphere cone of illumination (see Figure 6). Alignment of the two Type I windows is normally performed when they are suspended in the gantry opposite the windows.

e. Type II Window Simulator. The Type II window projects a 25-degree view. Figure 8 is a photograph of the Type II window and the celestial sphere assembly installed on the dolly and mount assembly. This window performs the same visual simulation function as that performed by the Type I windows. The image generators are, likewise, a black-and-white TV input projection system, a color TV input projection system, and a celestial sphere. Figure 9 is a schematic of the optical layout for the Type II window simulator.

f. Celestial Sphere Illumination System, Type II Window. The illumination system of the Type II window functions to generate the celestial sphere cone of illumination in a manner similar to that for the Type I windows.

F. Celestial Sphere

1. General

The celestial sphere (Figure 10) generates parallax-minimizing point source images, realistically simulating those of approximately one thousand stars, including selected stars of intensity as low as that of the 5th magnitude. The stellar images are extremely small, since the image generated by each ball is minified by the ratio of the distance of the light source over the focal distance (ball diameter/4) of the bearing ball. The illumination comprising each image is reflected by the celestial sphere beamsplitter (shown in Figure 6) into incidence with the celestial sphere mirror which projects the image illumination through the celestial sphere beamsplitter and TV take-off beamsplitter and corrector lens to the common focus. Illumination diverging from the common focus is reflected by the first beamsplitter into incidence with and collimation by the eyepiece mirror which projects the parallax-free star image through the exit pupil.

Thus it is apparent that the optical system is comprised of the features required to superimpose the separately generated TV images and celestial sphere images through the system exit pupil to a focus apparently at infinity.

The portion of the true celestial sphere observed at any instant depends on the attitude of the space capsule axis as described by the orthogonal, transient motions of pitch, roll, and yaw. Simulation of this changing view has been obtained by mounting the 27-inch-diameter celestial sphere in a three-gimbal axis system. The celestial pitch axis is normal to the simulated plane of the ecliptic; the celestial yaw axis is perpendicular to the pitch axis ecliptic plane, and the third axis (celestial roll) provides the so-called roll axis of rotation. The yaw axis is a yoke-type arrangement which supports the sphere and is itself rotated by the roll axis. The pitch drive is enclosed within the sphere.

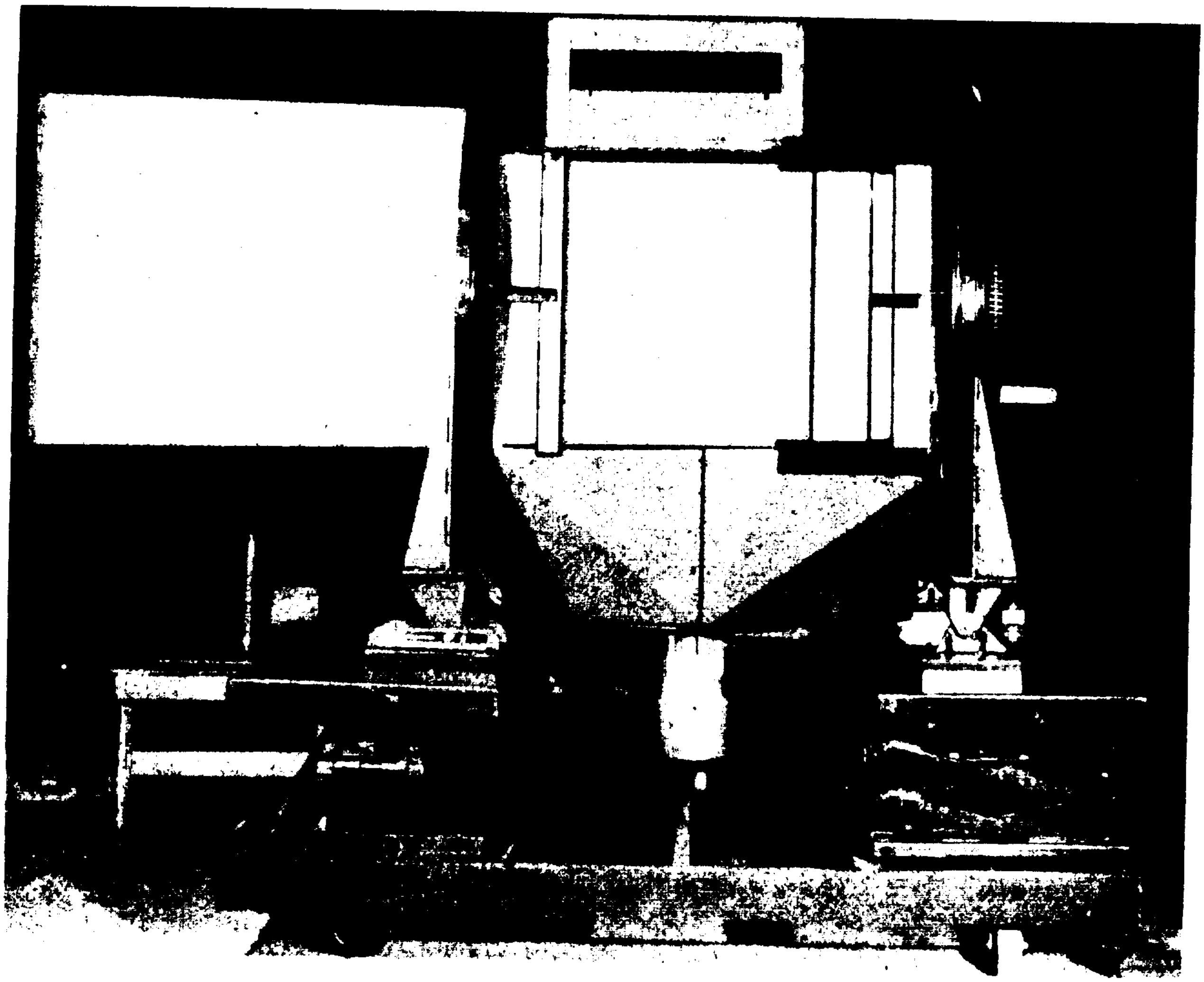

Figure 8. Type II Window and Celestial Sphere Assembly Installed on Dolly

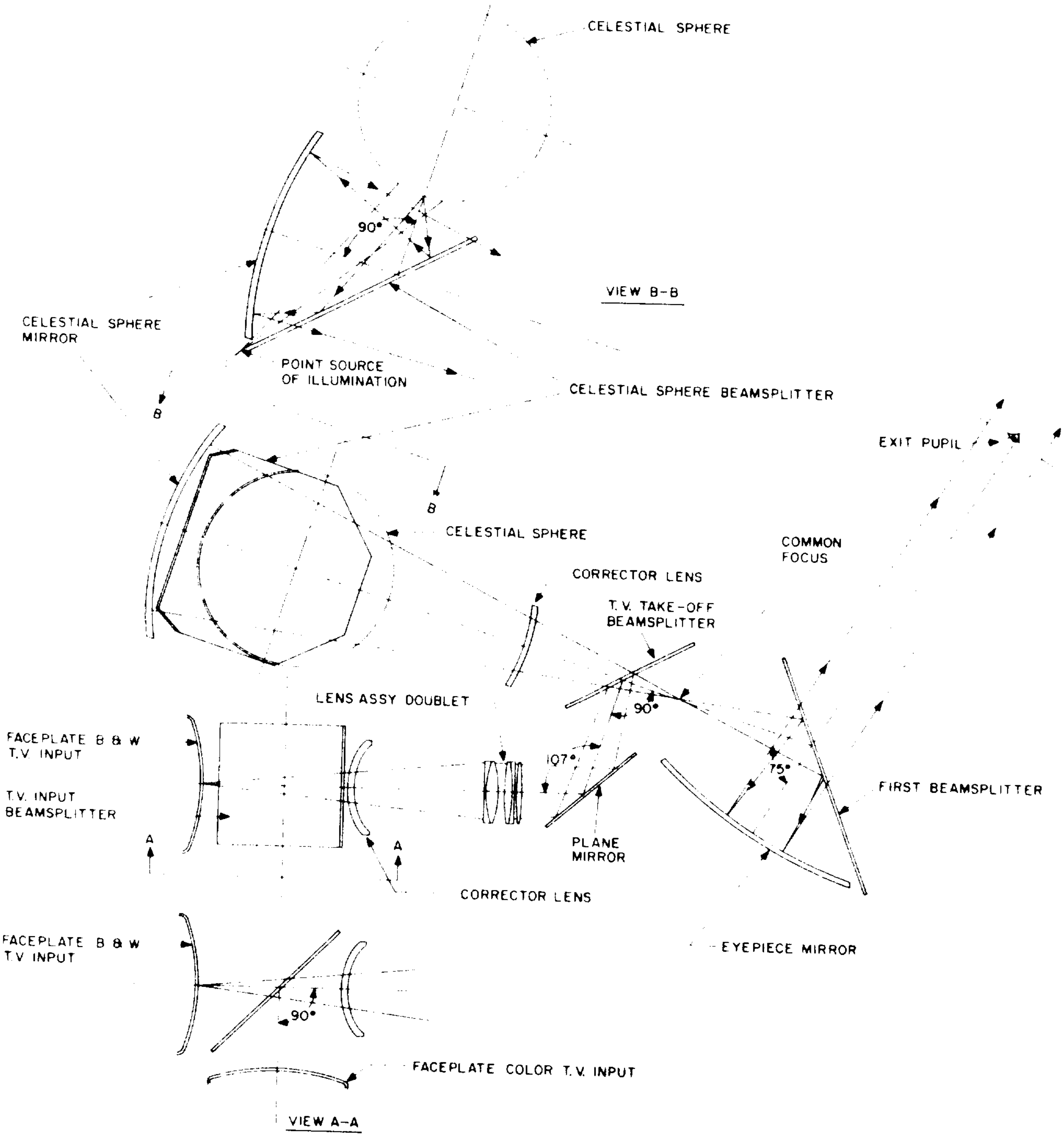

Figure 9. Optical Layout for Type II Window Simulator

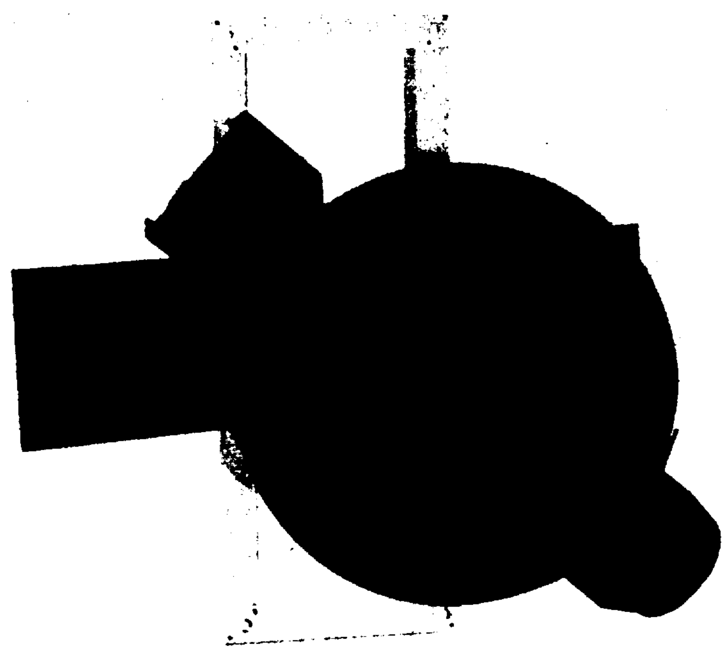

Figure 10. Celestial Sphere

2. Technical Data

a. General. The program-derived command signals are applied by synchro transmitters to the input terminals of the celestial sphere electronic system as shown in Figure 11. The system has three closed loop control channels, namely, the pitch (innermost) axis control loop; the yaw axis control loop, and the roll (outermost) control loop.

b. Pitch Axis Control Loop. Three-wire synchro data generated by the synchro transmitter located at the computer is coupled to the synchro receiver at the pitch axis of the celestial sphere. This 400-cycle error signal is amplified and demodulated. The output signal from this unit is a dc voltage whose amplitude is proportional to the amplitude of the ac error signal and whose polarity is dependent on the phase of the ac error signal. As shown in Figure 11, this dc signal is then applied to a passive stabilization network. The purpose of this network is to correct the servo characteristics of the control loop.

The dc tachometer located at the pitch axis provides an additional means of servo stabilization. This dc signal, proportional to velocity, is mixed with the dc-error signal at the preamplifier. Due to the high load inertia it was found desirable to provide tachometer damping whenever a large discrepancy exists between the command data and the actual position of the pitch axis. When this condition occurs (i.e., at initial "Turn-on") the maximum slewing velocity is limited by the tachometer signal, thereby reducing the amplitude and number of overshoots which would otherwise occur during the servo approach to the required null. The output from the dc preamplifier is next applied to a dc power amplifier. The system is phased so that the error signal causes the pitch motor to reposition the output shaft until the error signal is minimized.

c. Yaw Axis Control Loop. The yaw axis control loop is identical to that of the pitch axis except that two torque motors, mechanically and electrically connected in parallel, are used as the prime movers.

d. Roll Axis Control Loop. The roll axis control loop differs somewhat from that of the previously considered control loops. The high inertia (the roll axis carries both the pitch and yaw axes) required the use of a 300-watt torque motor utilizing a heavier duty power amplifier. In addition to power amplification this unit also incorporates dc preamplification, thereby eliminating the need for the separate preamplifier used in the pitch and yaw loops. The synchro command data is applied to the synchro receiver which is coupled to the roll axis in the same manner as that utilized in the pitch and yaw loops.

G. Window Support Structures

A variety of mounting positions for the optically functional windows is provided by the gantry assembly (see Figure 12). The gantry functions as a bridge upon the rails of which are mounted two identical end trucks; each end truck mounts a turntable. Three load chains on the hoists of each turntable are attached with a sling to the two Type I windows and the celestial sphere. A lateral drive is used to move the end truck rectilinearly along the gantry through a spur gear drive, and a rotary drive causes rotary motion of the turntable through activation of a circular worm drive.

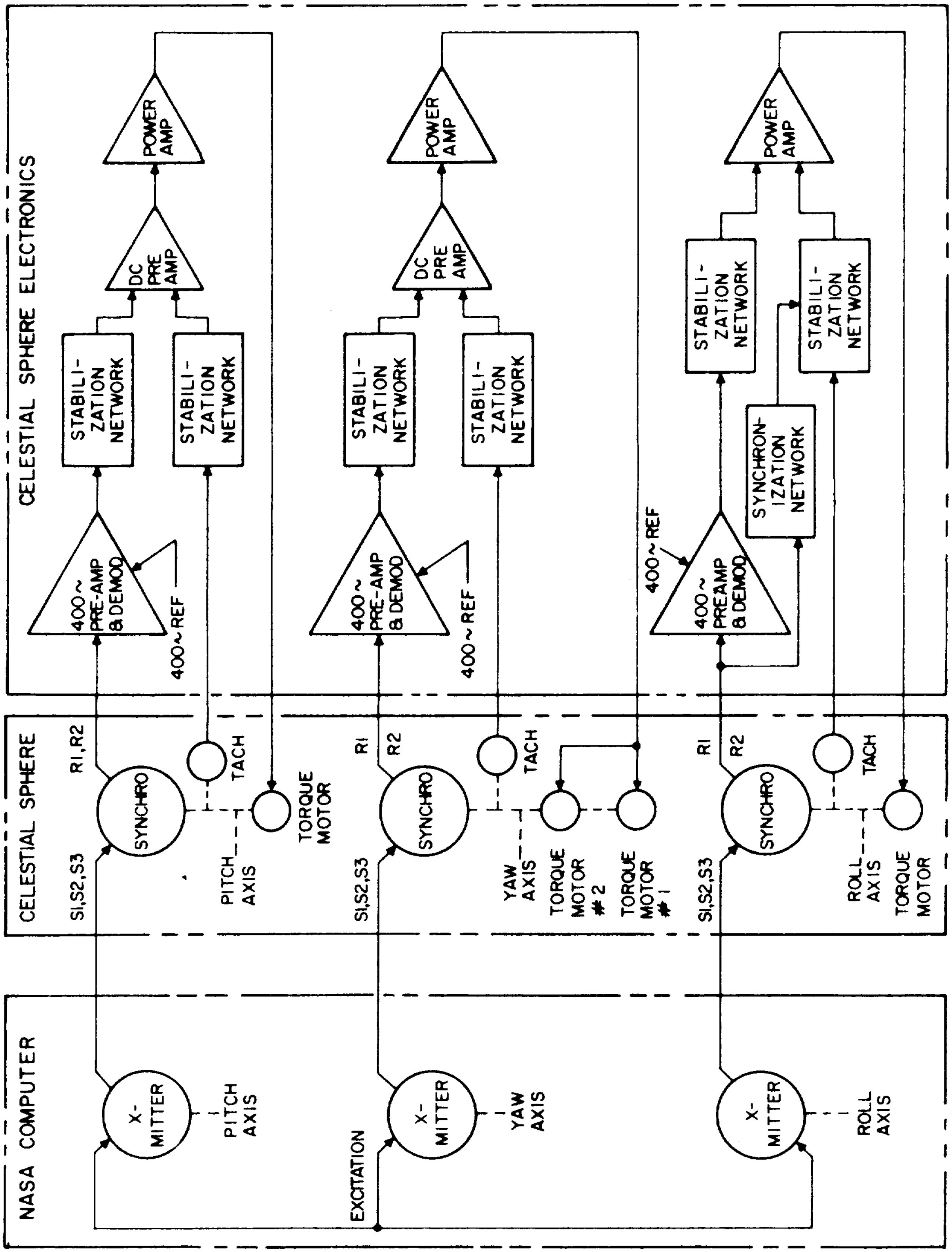

Figure 11. Celestial Sphere Electronic Block Diagram

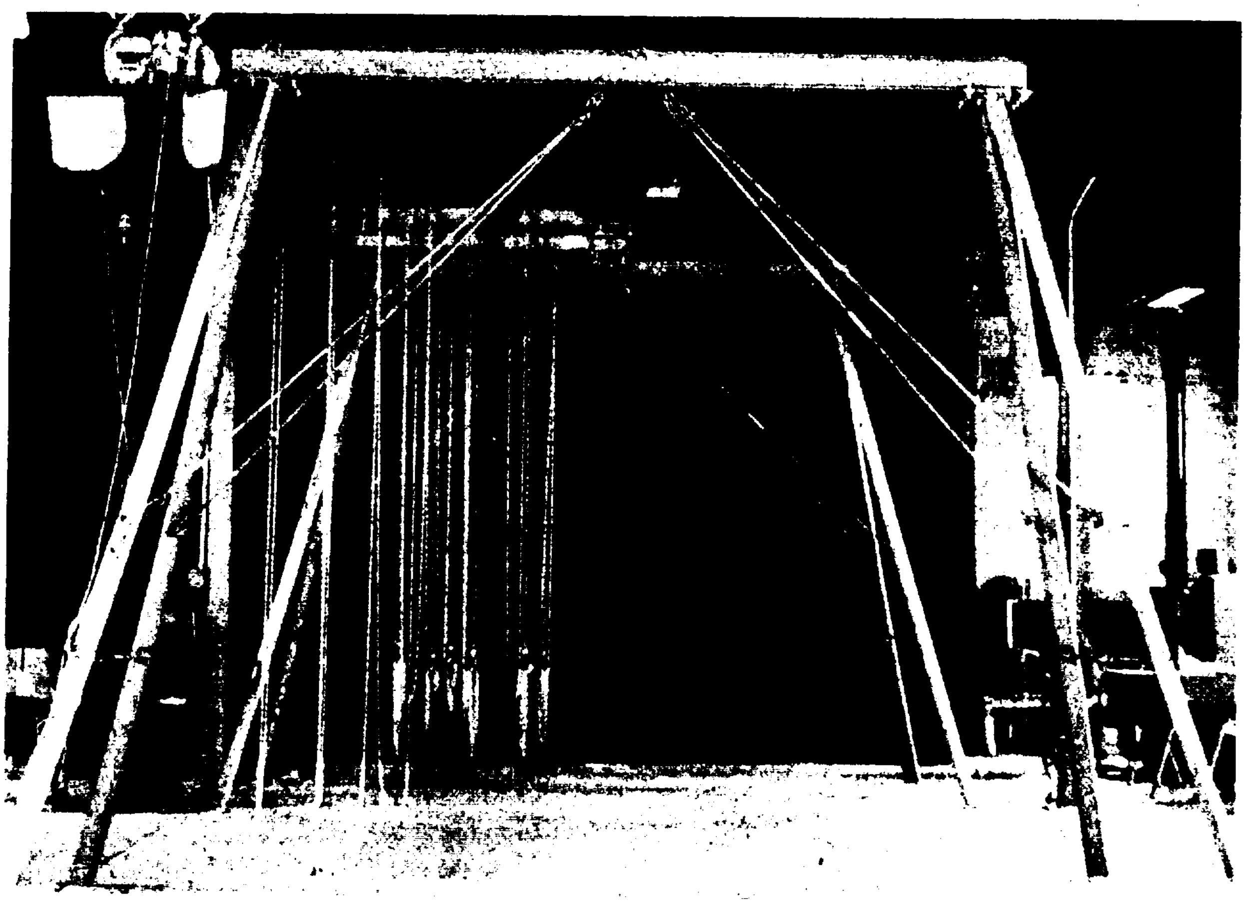

Figure 12. Gantry Assembly

Figure 13. Type II Window Installed on Dolly and Mount Assembly

2. Type II Window Mount and Dolly

The Type II window mount and dolly (shown in Figures 8 and 13) serves to align the optical window opposite the single view window of the trainer. The four-wheel dolly mounts two lift tables. A battery-energized tractor (Figure 13) serves to push or pull the dolly into position. Pushbutton controls are provided for adjusting elevation of the optical window in the vertical or any ±15-degree tilted plane. A rotational mechanism (shown in Figure 13) consisting of a brake, rotational worm drive, index plates and stops, permits rotation of the Type II window about its axis of rotation.

APPENDIX
CORDIC ERROR STUDY

1. Description of CORDIC

The CORDIC (coordinate rotation digital computer) algorithm is a computational method for finding the components of a point or vector after coordinate rotation.* It is useful in problems involving conversion from one coordinate system to another, and conversion from rectangular to polar coordinates, and polar to rectangular coordinates.

Consider a coordinate system XYZ and a vector $V_0 = (X_0, Y_0, 0)$. Rotation of the coordinate system through an angle $-\theta$ about its Z axis is mathematically the same as rotating V_0 about the Z axis through the angle $+\theta$. Because rotation of the vector is easier to draw, suppose that we want to rotate V_0 into V_1 through the angle θ. See Figure A-1. The components of the new vector $V_1 = (X_1, Y_1, 0)$ are given by

$$X_1 = X_0 \cos\theta - Y_0 \sin\theta$$

$$Y_1 = Y_0 \cos\theta + X_0 \sin\theta$$

The computation is usually carried out by use of extensive tables of sines and cosines or by evaluation of power series for sine and cosine.

Now suppose that instead of carrying the head of the vector V_0 through an arc, it is constrained to follow a straight path as shown in Figure A-1. A vector V_1' is obtained, which is collinear with V_1 but too long by a factor sec θ. It does, however, have a simpler relationship to V_0. ΔV, being perpendicular to V_0 and having a magnitude tan θ times the magnitude of V_0, must have the components $(-Y_0 \tan\theta, X_0 \tan\theta, 0)$. Therefore, since $V_1' = V_0 + \Delta V$,

$$X_1' = X_0 - Y_0 \tan\theta$$

$$Y_1' = Y_0 + X_0 \tan\theta$$

Striving for still greater simplicity, next assume θ to be such that its tangent is a negative power of 2, say $\tan\theta = 2^{-k}$. In binary notation, multiplication by 2^{-k} is accomplished by shifting the number k places to the right (toward the least significant end). The transformation equations become

$$X_1' = X_0 \mp 2^{-k} Y_0$$

$$Y_1' = Y_0 \pm 2^{-k} X_0$$

where the signs are chosen in accordance with the sign of θ.

*This technique was developed by J. E. Volder at Convair, Division of General Dynamics Corp. See Volder, J. E., "CORDIC Trigonometric Computing Technique," IRE Trans. on Electronic Computers, EC 8, September 1959.

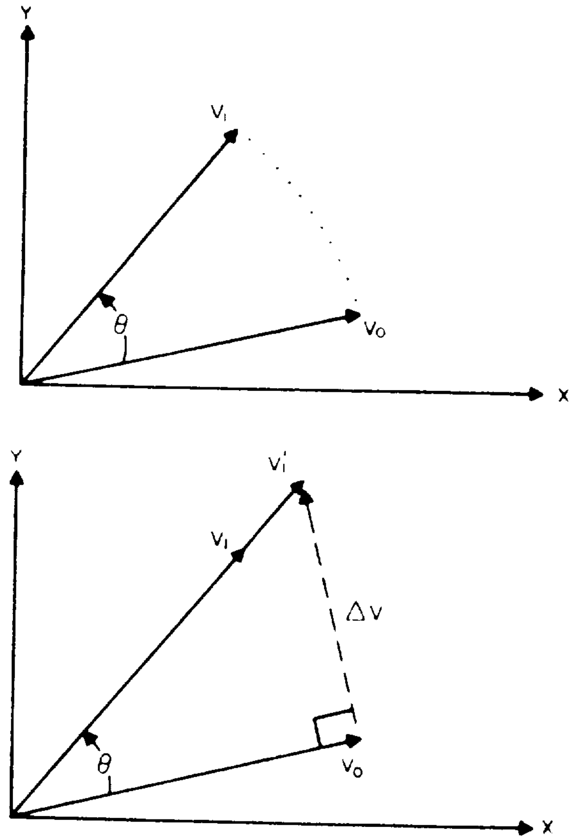

Figure A-1

The final step makes use of the fact that by proper choice of signs, any angle θ can be expressed as

$$\theta = \pm 90^{\circ} \pm \arctan 1 \pm \arctan 1/2 \pm \ldots \pm \arctan 2^{-n}$$

to an accuracy of at least $\pm \arctan 2^{-n-1}$. This is called its arctangent radix representation. Rotation through ±90 degrees involves only interchanging components and adjusting signs:

$$X_1 = \mp Y_0$$

$$Y_1 = \pm X_0$$

Thus suppose the arbitrary angle

$$\theta = a_0\, 90^{\circ} + a_1\, 45^{\circ} + a_2 \arctan 1/2 + \ldots + a_{n+1} \arctan 2^{-n}$$

where the a's are all ±1. We want to rotate the vector $V = (X, Y, 0)$ through this angle. The algorithm goes as follows:

$$X_0 = -a_0 Y$$

$$Y_0 = +a_0 X$$

$$X_1 = X_o - a_1 Y_o$$

$$Y_1 = Y_o + a_1 X_o$$

$$X_2 = X_1 - (1/2) a_2 Y_1$$

$$Y_2 = Y_1 + (1/2) a_2 X_1$$

.

.

.

$$X_{n+1} = X_n - a_{n+1} 2^{-n} Y_n$$

$$Y_{n+1} = Y_n + a_{n+1} 2^{-n} X_n$$

This algorithm requires only addition/subtraction and shifting. The result, $V' = (X_{n+1}, Y_{n+1}, 0)$ is collinear with the desired vector but has a magnitude too large by the factor

$$K = \prod_{i=0}^{n} \sec \arctan 2^{-i} = \prod_{i=0}^{n} (1 + 2^{-2i})^{\frac{1}{2}}$$

K is called the CORDIC constant and is approximately equal to 1.65. Since the same sequence of part rotations is followed every time (with only the a's changing), K is a function only of n; it can be computed and stored in memory. When necessary the components X_{n+1}, Y_{n+1} can be multiplied by K^{-1} to yield the components of the desired vector.

For a word length of 20 bits the constants used are shown in Table A-1.

As an illustration consider the following inputs:

$X = 1.7321$	0 0 1 . 1 0 1 1 1 0 1 1 0 1 1 0 1 0 1 0 1	
$Z = -1.00$	1 1 1 . 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	
$\theta \doteq 60^o$	1 1 0 . 1 0 1 0 1 0 1 0 1 0 0 0 0 0 0 0 0	

The CORDIC algorithm is shown for these inputs by Table A-2.

2. Error Study

For n-bit input quantities, the CORDIC algorithm is performed with n-bit accuracy. Like single precision (single word length) multiplication, this requires rounding the addend and subtrahend quantities. Unlike single precision multiplication, however, the resulting rounding errors are propagated. That is, the rounding error enters into later iterations and produces further errors. This error propagation does not lend itself to a closed analysis.

In a machine employing the CORDIC technique, it is important to know the magnitude of the error in order to determine word lengths. For this reason, an experimental investigation of roundoff error with the CORDIC algorithm was conducted.

TABLE A-1

1) Reciprocal of the CORDIC constant

0 . 1 0 0 1 1 0 1 1 0 1 1 1 0 1 0 0 1 1 1

2) Binary Arctan Constants (ATR constants)

BINARY ARCTAN

ATR 1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ATR 2	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ATR 3	0	0	0	1	0	0	1	0	1	1	1	0	0	1	0	0	0	0	0	0
ATR 4	0	0	0	0	1	0	0	1	1	1	1	1	1	0	1	1	0	0	1	1
ATR 5	0	0	0	0	0	1	0	1	0	0	0	1	0	0	0	1	0	0	0	1
ATR 6	0	0	0	0	0	0	1	0	1	0	0	0	1	0	1	1	0	0	0	0
ATR 7	0	0	0	0	0	0	0	1	0	1	0	0	0	1	0	1	1	1	0	1
ATR 8	0	0	0	0	0	0	0	0	1	0	1	0	0	0	1	0	1	1	1	1
ATR 9	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	1	0	1	1	1
ATR 10	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	1	0	1	1
ATR 11	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	1	0	1
ATR 12	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	1	0
ATR 13	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	1
ATR 14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0
ATR 15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0
ATR 16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
ATR 17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
ATR 18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
ATR 19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

TABLE A-2

	Y REGISTER	X REGISTER	ANGLE
			11010101010100000000
	00110111011011010101	11100000000000000000	01000000000000000000
1	00100000000000000000	00110111011011010101	00010101010100000000
	00110111011011010101	11100000000000000000	11100000000000000000
2	01010111011011010101	00010111011011010101	11110101010100000000
	11110100010010010101	00101011101101101011	00010010111001000000
3	01001011101101101010	01000011001001000000	00001000001101000000
	00010000110010010000	11101101000100100101	11110110000001001101
4	01011100011111111010	00110000001101100101	11111110001110001101
	11111001111110010100	00001011100011111111	00000101000100010001
5	01010110011110001110	00111011110001100100	00000011010010011110
	00000011101111000110	11111010100110000111	11111101011101010000
6	01011010001101010100	00110110010111101011	00000000101111101110
	00000001101100101111	11111101001011100101	11111110101110100011
7	01011011111010000011	00110011100011010000	11111111011110010001
	11111111001100011101	00000001011011111010	00000000101000101111
8	01011011000110100000	00110100111111001010	00000000000111000000
	00000000011010100000	11111111010010011101	11111111101011101001
9	01011011100001000000	00110100010001100111	11111111110010101001
	11111111110010111100	00000000010110111000	00000000001010001011
10	01011011010011111100	00110100101000011111	11111111111100110100
	11111111111001011011	00000000001011011010	00000000000101000101
11	01011011001101010111	00110100110011111001	00000000000001111001
	00000000000011010011	11111111111010010011	11111111111101011110
12	01011011010000101010	00110100101110001100	11111111111111010111
	11111111111110010111	00000000000010110111	00000000000001010001
13	01011011001111000001	00110100110001000011	00000000000000101000
	00000000000000110101	11111111111110100101	11111111111111011000
14	01011011001111110110	00110100101111101000	00000000000000000000
	00000000000000011010	11111111111111010010	11111111111111101100
15	01011011010000010000	00110100101110111010	11111111111111101100
	11111111111111110011	00000000000000010111	00000000000000001010
16	01011011010000000011	00110100101111010001	11111111111111110110
	11111111111111111001	00000000000000001011	00000000000000000101
17	01011011001111111100	00110100101111011100	11111111111111111011
	11111111111111111101	00000000000000000110	00000000000000000010
18	01011011001111111001	00110100101111100010	11111111111111111101
	11111111111111111111	00000000000000000010	00000000000000000001
19	01011011001111111000	00110100101111100100	11111111111111111110

KEY: $\underline{1}$ = DATA AS IT WOULD APPEAR AT END OF FIRST ITERATION

$\underline{2}$ = DATA AS IT WOULD APPEAR AT END OF SECOND ITERATION

ETC.

The CORDIC algorithm was programmed for the IBM 1620 computer. (Figure A-2 summarizes the 1620 program used in this study.) Paralleling this was a check computation described by the following equation

$$X_1 = X_o \cos\theta - Y_o \sin\theta$$

$$Y_1 = Y_o \cos\theta + X_o \sin\theta$$

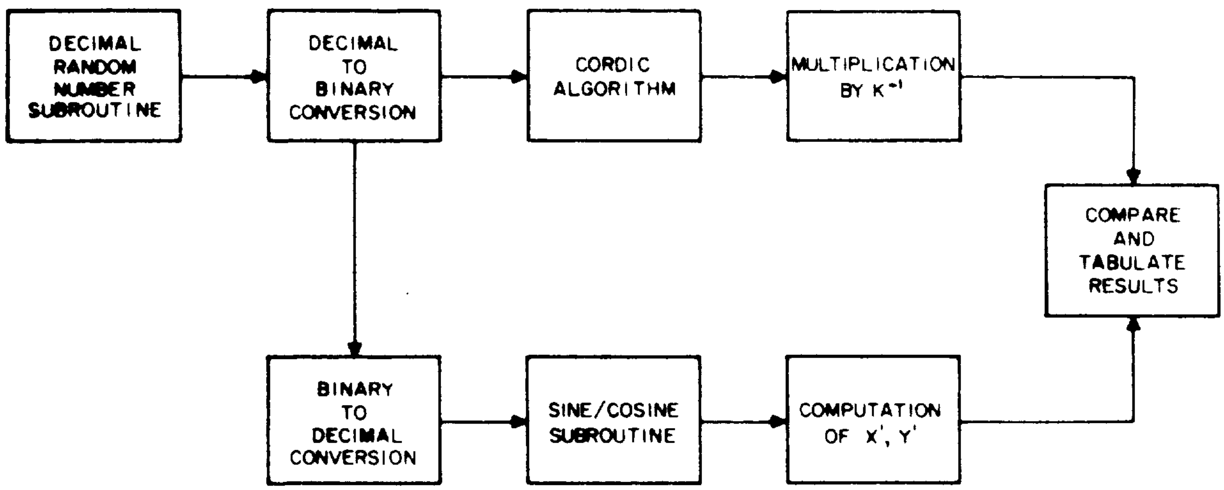

Figure A-2. IBM 1620 Program for CORDIC Error Study

This computation was carried out with a resolution greatly in excess of that of the CORDIC computation. Thus the check computation could be considered error-free for purposes of this investigation.

Inputs (X, Y, θ) were provided by a random number subroutine. Outputs of the CORDIC and check subroutines were compared and the CORDIC errors tabulated.

The random number subroutine is based on a technique described by J. Certaine.* The sine/cosine subroutine employs evaluation of a series for sine and cosine. The coefficients were computed to 30 decimal places.

The object of the investigation was the error distribution function (or equivalently, the error frequency density function). The number of cases necessary to estimate the error distribution function was calculated assuming the Kolmogorov-Smirnov

*J. Certaine, "On Sequences of Pseudorandom Numbers of Maximal Length," Association for Computing Machinery Journal, 5, 1958, p. 353.

TABLE A-3a

ERROR DISTRIBUTION FOR X FOR N = 16

10101	00000
10110	00000
10111	00000
11000	00000
11001	00000
11010	00000
11011	00000
11100	00020
11101	00239
11110	01440
11111	04170
00000	04638
00001	02314
00010	00474
00011	00048
00100	00002
00101	00000
00110	00000
00111	00000
01000	00000
01001	00000
01010	00000
01011	00000

TABLE A-3b

ERROR DISTRIBUTION FOR Y FOR N = 16

10101	00000
10110	00000
10111	00000
11000	00000
11001	00000
11010	00000
11011	00000
11100	00010
11101	00197
11110	01391
11111	04008
00000	04932
00001	02283
00010	0047[illegible]
00011	00050
00100	[illegible]3
00101	[illegible]1
00110	[illegible]
00111	[illegible]
01000	00000
01001	00000
01010	[illegible]
01011	[illegible]

TABLE A-4a

ERROR DISTRIBUTION FOR X FOR N = 20

10101	00000
10110	00000
10111	00000
11000	00000
11001	00000
11010	00000
11011	00013
11100	00156
11101	00909
11110	03397
11111	07318
00000	08302
00001	04882
00010	01546
00011	003[illegible]9
00100	00039
00101	0000[illegible]
00110	00000
00111	00000
01000	00000
01001	00000
01010	00000
01011	00000

TABLE A-4b

ERROR DISTRIBUTION FOR Y FOR N = 20

10101	00000
10110	00000
10111	00000
11000	00000
11001	00000
11010	00002
11011	00014
11100	00145
11101	00847
11110	03432
11111	07183
00000	08540
00001	04831
00010	01523
00011	00316
00100	00040
00101	00003
00110	00000
00111	00000
01000	00000
01001	00000
01010	00000
01011	00000

TABLE A-5a

ERROR DISTRIBUTION FOR X FOR N = 26

10101	00000	*
10110	00000	*
10111	00000	*
11000	00000	*
11001	00000	*
11010	00003	*
11011	00012	*
11100	00093	*
11101	00372	*
11110	01167	*
11111	02104	*
00000	02310	*
00001	01547	*
00010	00620	*
00011	00176	*
00100	00031	*
00101	00004	*
00110	00001	*
00111	00000	*
01000	00000	*
01001	00000	*
01010	00000	*
01011	00000	*

TABLE A-5b

ERROR DISTRIBUTION FOR Y FOR N = 26

10101	.00000
10110	00000
10111	00000
11000	00000
11001	00000
11010	[illegible]
11011	00007
11100	00083
11101	.00380
11110	.01162
11111	.02111
00000	.02334
00001	.01535
00010	.00621
00011	.00169
00100	.00034
00101	[illegible]
00110	.00001
00111	.00000
01000	.00000
01001	[illegible]
01010	[illegible]
01011	00000

one-sample test.* Taking a level of confidence of 0.01, the width of the confidence belt for the distribution function is given by

$$D = \pm \frac{1.63}{\sqrt{N}}$$

where N is the number of cases. Following is a list of the three experimental runs (each for a given number of bits), the number of cases used and the width of the confidence belt at a level of confidence of 0.01.

No. Bits	No. Cases	Width of Confidence Belt
20	26,876	less than ±0.01
16	13,350	less than ±0.015
26	8,440	less than ±0.018

Since the 20-bit word length was contemplated for the machine to be built, more accuracy was desired for this case.

The results of these three runs are presented in Tables A-3, A-4, and A-5. For each run (a) the frequency density function for X, and (b) the frequency density function for Y are given.

The CORDIC errors have two sources. First, the conversion of the input angle to arctangent radix representation with a fixed number of digits is necessarily an approximation. Second, the cross-addition process involves rounding off addends. The second error is propagated by the algorithm while the first is not. The second error is in general the more important.

*S. Siegel, Nonparametric Statistics, New York: McGraw-Hill, 1956, p. 47.